Discovery School Developmental Continuum for Mathematics

Preconventional Ages 3-5	Emerging Ages 4-6	Developing Ages 5-7	Beginning Ages 6-8	Expanding Ages 7-9
① Counts small sets of objects, actions and sounds using one-to-one correspondence. ① Counts verbally up to five. ① Counts verbally backwards from five using objects, and actions. ① Begins to recognize numbers 1 to 5. ① Begins to recognize language of more or less relating to sets of objects. ① Recognizes 1st –3rd ↔ Recognizes temperature differences in seasons: fall, winter, spring, summer ↔ Uses non-standard measuring units (body parts, unifix cubes, manipulatives) ↔ Begins to recognize the different types of coins (penny, nickel, dime) ↔ Uses measuring tools for volume in cooking projects ❑ Recognizes and names basic geometric shapes in the environment ❑ Matches shapes & objects ❑ Sorts and compares geometric shapes with guidance ✘ Recognizes, and copies patterns such as sequences of sounds, shapes, color – ABAB, AAB with guidance ✘ Sorts objects and events. ✓ Participates in open-ended questions related to gathering data with guidance ✓ Makes simple concrete graphs reflecting opinions and events with guidance	① Counts and recognizes numbers 0-20 by reading and writing them ① Recognizes if small sets of objects are same or different; more or less ① Counts backwards from 10 using number lines & finger plays ① Counts on, by one, from any point (2 to 19) ① Solves simple addition and subtraction, and fair-share type problems with the use of manipulatives ① Understands and demonstrates that the arrangement of objects within a set does not effect the quantity ① Recognizes and names the different types of coins (penny, nickel, dime) with guidance ↔ Organizes the day into the time frames of morning, afternoon, and night ↔ Develops language of measurement such as bigger, longer, shorter, lighter ↔ Participates in group counting and marking during calendar time ↔ Explores the measurement variables of length, volume, mass and weight ↔ Develops the concept of time as measured by months, weeks, and days ❑ Sorts, compares, and classifies geometric shapes and other objects by attributes (shape, size, and color) ❑ Uses vocabulary of side, angle, and corner to describe shapes with guidance ❑ Describes the relative position of above, below, left, right, over, under with guidance ✘ Recognizes, copies, extends patterns: sequences of sounds, shapes, position, color, and number with guidance ✘ Sorts, classifies, and orders objects and events ✘ Uses symbols for addition and subtraction ✓ Demonstrates ability to pose questions and gather data; Participates in gathering data about opinions and events ✓ Uses concrete objects, pictures, and graphs to represent data ✓ Describes data by using middle, most, least and same with guidance	① Adds and subtracts numbers up to 20 ① Develops an understanding that adding and subtracting are inverse operations ① Develops an understanding of the base 10 numeration system to 100 ① Recognizes whole numbers to 100 in terms of groups of 10's and 1's ① Orders, compares and skip counts to 20 independently ① Uses symbols ($, ¢) correctly up to $100 with guidance ↔ Estimates and compares length/heights of objects with non-standard units ↔ Measures to nearest inch, cm independently ↔ Tells time to the hour, ½ hour with guidance ↔ Names and numbers the months with guidance ❑ Makes 2 dimensional shapes on geoboards with guidance ❑ Constructs polygons from other shapes with guidance ❑ Identifies and draws symmetrical polygons by the # of sides with guidance ✘ Recognizes, copies, extends, and creates patterns such as sequences of sounds, shapes, position, color, and number independently ✘ Verbalizes the process of change as reflected in the change of the seasons ✓ Reads graphs ✓ Makes tally marks to organize data ✓ Answers questions using a graph: What happens most? Where is the middle? ✓ Describes data by using middle, most, least and same ✓ Discriminates between impossible, probable & real world events with guidance	① Counts, reads and writes numbers to 100 fluently ① Adds and subtracts whole numbers to 100 w/o regrouping/renaming independently ① Uses concept of base-ten numeration including counting in units and multiples of hundreds, tens, and ones to 1000 with guidance ① Recognizes place value in expanded notation to 1000 ① Orders, compares and skip counts to 1000 independently ① Uses symbols ($, ¢) correctly up to $100 independently ① Identifies basic fractions 1/3, 2/3, 1/8, ¼, ½, ¾ with guidance ① Carries and borrows with 3 digit numbers with guidance ↔ Estimates and compares differences in height and length of objects with standard units with guidance ↔ Measures to nearest ½ inch and ½ centimeter independently ↔ Tells time to the nearest 5 minutes independently ↔ Names and orders the months of year independently ❑ Makes 2 dimensional shapes on geoboards independently ❑ Uses geometric knowledge for understanding area, fractions and proportion with guidance ❑ Identifies and constructs polygons up to 9 sides with guidance ❑ Identifies symmetrical polygons independently ✘ On a number grid, identifies numbers, patterns and counts forward and back ✘ Finds patterns in +/- facts ✓ Collects data by counting and interviewing ✓ Collects, reads and interprets data from print, posters, maps, charts etc. with guidance ✓ Makes a bar graph, table and chart with guidance ✓ Discriminates between impossible, probable & real world events	① Reads and writes numbers to 10,000 ① Develops fluency with basic add /subtract facts ① Uses concept of base-ten numeration including counting in units and multiples of hundreds, tens, and ones to 1000 independently ① Uses strategies (number lines, fact grid) to solve addition and subtraction problems ① Understands concept of basic fractions ⅓ ⅔ ⅛ ¼ ½ ¾ with the use of manipulatives ① Understands the size of a fractional part is relative to the size of the whole, and that fractions represent numbers that are equal to, less than, or greater than 1 ① Finds equivalent fractions with the use of manipulatives ① Solves multiplication and division number stories or problems through use of representations - equal-sized groups, arrays, area models, and equal "jumps" on number lines for multiplication, and successive subtraction, partitioning, and sharing for division ↔ Understands relationship of metric units of measurement and the power of ten ↔ Measures linear, volume + temperature using metric and standard units with greater precision ❑ Creates symmetrical figures and transformations with guidance ❑ Describes, analyzes, compares, and classifies two dimensional shapes by sides and angles, and congruency with guidance ❑ Solves for area and perimeter of two dimensional shapes with guidance ✘ Identifies growing and repeating patterns with guidance ✘ Explores patterns in doubling numbers and square numbers ✘ Writes and solves number sentences with missing variables with guidance ✓ Reads and interprets data in tables, graphs, and maps ✓ Collects data from print, posters, maps, charts etc. with guidance ✓ Begins to create bar and circle graphs from raw data to explain meaning with guidance

① = Number and Operations ↔ = measurement ❑ = geometry ✘ = Algebra ✓ = Data Analysis

Discovery School Developmental Continuum for Mathematics

Bridging Ages 8-10	Fluent Ages 9-11	Proficient Ages 10-13	Connecting Ages 11-14	Independent
① Uses vocabulary to explain math sentences ① Is fluent in adding and subtracting numbers through 10,000 place independently ① Expands understanding of division to two digits in the dividend ① Applies rules of divisibility to whole division numbers and fractions with guidance ① Adds and subtracts (with or without regrouping) decimals using money as a model ① Develops quick recall of multiplication facts and related division facts ① Understands the relationship between fractions and decimals ① Begins to determine factors and multiples of whole numbers ① Compares and orders fractions with models ↔ Understands perimeter as distance around an object with guidance ↔ Uses formula for finding area and perimeter of: two dimensional quadrilaterals with guidance ❑ Creates symmetrical figures and transformations, independently ❑ Describes, analyzes, compares, and classifies two dimensional shapes by sides, angles, and congruency ❑ Measures angles (acute, obtuse, and right) in the environment with guidance ✘ Identifies growing and repeating patterns independently ✘ Writes and solves number sentences with one missing variable independently ✓ Collects data from print, posters, maps, charts etc. ✓ Creates and analyzes frequency tables: bar graphs, picture graphs, line plots and uses them to solve problems with guidance ✓ Creates an X and Y plot to record data with guidance	① Develops fluency with whole number multiplication and division to million with guidance ① Compares and orders fractions without models ① Determines what is an integer ① Compares and orders integers, including negative numbers, such as below sea level or temperature ① Adds and subtracts integers ① Identifies prime and composite numbers ① Determines factors and multiples of whole numbers with guidance ① Employs problem solving strategies: i.e. using tables, guess and check, etc. ↔ Understands perimeter as distance around an object independently ↔ Uses formula for finding area and perimeter of: quadrilaterals, triangles, circles ↔ Finds surface area of prisms and cylinders ↔ Finds volume of prisms ❑ Measures angles independently ❑ Discerns congruent and similar figures with guidance ❑ Constructs and measures angles with guidance ❑ Analyzes properties of polyhedral solids, edges, faces, vertices with guidance ✘ Solves algebraic problems with guidance ✘ Identifies properties of integers with guidance ✘ Evaluates and simplifies mathematical expressions with guidance ✘ Finds Greatest Common Factor and Least Common Multiple with guidance ✓ Creates an X and Y plot to record data ✓ Constructs and interprets bar graphs, line graphs, circle graphs with guidance ✓ Chooses an appropriate graph for given data ✓ Identifies mean, median, mode and range within a data set with guidance ✓ Constructs and interprets frequency tables, line plots with guidance	① Develops fluency with whole number multiplication and division ① Identifies square numbers with guidance ① Writes mathematical expressions using exponents with guidance ① Determines factors and multiples of whole numbers ① Finds Greatest Common Factor and Least Common Multiple ① Evaluates word problems ↔ Converts within standard system (ft to yds) ↔ Uses metric system (cm, m, km, etc.) ↔ Converts within metric system (m. to km.) ↔ Chooses appropriate unit for what is being measured. ↔ Applies concepts of units of time and elapsed time ❑ Identifies: points, lines, rays, planes, segments, collinear/ noncollinear ❑ Discerns congruent and similar figures and lines of symmetry independently ❑ Constructs and measures angles independently ❑ Analyzes properties of polyhedral solids, edges, faces, vertices ✘ Identifies properties of integers ✘ Determines absolute value of integers; +/- integers; compares integers ✘ Solves two step equations with guidance ✘ Develops understanding of and fluency with multiplication and division of integers with guidance ✘ Solves equations with integers with guidance ✘ Applies order of operations to problems ✘ Evaluates and simplifies mathematical expressions independently ✓ Constructs and interprets bar graphs, line graphs, circle graphs ✓ Identifies mean, median, mode and range within a data set independently ✓ Constructs and interprets frequency tables, line plots independently ✓ Identifies and explains misleading graphs	① Demonstrates fluency with multiplication and division of fractions and decimals ① Uses common procedures to multiply and divide fractions and decimals efficiently and accurately including multi-step problems involving measurement. ① Identifies square numbers ① Writes mathematical expressions using exponents ↔ Uses formula for finding area of trapezoids ↔ Applies multiple formulas to find area of composite figures ↔ Finds volume and surface area of cylinders ↔ Explains and uses Pythagorean Theorem ❑ Constructs segments, parallel lines, and perpendicular lines ❑ Explains the Pythagorean Theorem and solves problems with right triangles ✘ Identifies properties of integers ✘ Determines absolute value of integers; +/- integers; compares integers independently ✘ Solves two step equations independently ✘ Uses variables to represent numbers whose exact values are not yet specified ✘ Solves equations with integers ✓ Constructs and interprets: tables, charts, bar graphs, line graphs, circle graphs independently ✓ Chooses an appropriate graph or given data ✓ Constructs: Histograms, Double bar graphs, Double line Graph, Scatter plots/ Correlation, Stem and leaf plots, Identifies outliers	

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